POSITIVE SOLUTIONS OF DIFFERENCE BOUNDARY VALUE PROBLEMS WITH MIXED-MONOTONICITY*

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Abstract. The existence of positive solutions are studied for second-order difference boundary value problems with mixed-monotonicity or semipositone mixed-monotonicity. By using fixed point index theorem, the new existence theorems are achieved, which improve the related results with positone nonlinearity. Our results are demonstrated respectively by two examples.

Keywords. Difference boundary value problem, positive solution, Green's function, fixed point index, cone.

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1 Introduction

Many problems in applied mathematics lead to the study of the difference system. Recently, many attentions have been paid to the existence of positive solutions of monotone difference boundary value problems, see [3,4]. However, very few work has been done to the existence of positive solutions for mixed-monotone difference boundary value problems, especially the existence of positive solutions for semipositone mixed-monotone difference boundary value problems.

The purpose of this paper is to study the following difference boundary value problem:

$$\begin{cases} \Delta^2 u(k) + f(k, u(k)) = 0, \quad k \in [0, T] \\ u(0) = u(T+2) = 0 \end{cases} \tag{1}$$

where $T > 1$ is a fixed positive integer, $\Delta u(k) = u(k+1) - u(k)$, $\Delta^2 u(k) = \Delta(\Delta u(k))$ and $[a, b] = \{a, a+1, \cdots, b\} \subset Z$ the set of all integers. A positive solution of (1) we mean a solution of (1) which is nonnegative and nontrivial. Our aim is to establish some results for the existence of positive solutions