Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **16 (2009)** 27-40
 http://www.watam.org

GLOBAL EXISTENCE AND UNIQUENESS RESULTS FOR IMPULSIVE FUNCTIONAL DIFFERENTIAL EQUATIONS WITH VARIABLE TIMES AND MULTIPLE DELAYS

John R. Graef[1] and Abdelghani Ouahab[2]

[1]Department of Mathematics
University of Tennessee at Chattanooga
Chattanooga, TN 37403-2598 USA
[2]Department of Mathematics
University of Sidi Bel Abbes
BP 89 2000 Sidi Bel Abbes, Algeria

Abstract. In this paper, we give global existence and uniqueness results for first order impulsive functional differential equations with variable times and multiple delays. We rely on a recent nonlinear alternative of Leray-Schauder type in Fréchet spaces due to Frigon and Granas.
Keywords. Impulsive functional differential equations, existence, uniqueness, multiple delays, variable times, Fréchet spaces.
AMS (MOS) subject classification: 34A12, 34A34, 34A37, 34K45.

1 Introduction

This paper is concerned with the existence and uniqueness of solutions of the first order impulsive differential equation with variable moments and multiple delays

$$y'(t) = f(t, y_t) + \sum_{i=1}^{p} y(t - T_i), \quad \text{a.e. } t \in J := [0, \infty), \quad t \neq \tau_k(y(t)), \tag{1}$$

$$y(t^+) - y(t^-) = I_k(y(t)), \quad t = \tau_k(y(t)), \tag{2}$$

$$y(t) = \phi(t), \quad t \in [-r, 0], \tag{3}$$

where $k \in \mathbb{N}$, $0 < r < \infty$, $\tau_1(y(t)) < \tau_2(y(t)) < \ldots$, $\lim_{n\to\infty} \tau_n(y(t)) = \infty$,

$$\mathcal{D} = \{\psi : [-r, 0] \to \mathbb{R}^m \mid \psi \text{ is continuous everywhere except for a countable number of points } \bar{t} \text{ at which } \psi(\bar{t}^-) \text{ and } \psi(\bar{t}^+) \text{ exist}, \psi(\bar{t}^-) = \psi(\bar{t}), \text{ and } \sup_{\theta\in[-r,0]} |\psi(\theta)| < \infty\},$$

$f : J \times \mathcal{D} \to \mathbb{R}^m$, and $I_k \in C(\mathbb{R}^m, \mathbb{R}^m)$, $k = 1, 2, \ldots$, $\tau_k : \mathbb{R}^m \to \mathbb{R}$, $k = 1, 2, \ldots$, and $\phi : [-r, 0] \to \mathbb{R}^m$ are continuous, and $0 \leq T_i \leq r$ for