

## SYNCHRONIZATION PROBLEMS FOR UNIDIRECTIONAL FEEDBACK COUPLED NONLINEAR SYSTEMS

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**Abstract.** In this paper we consider three different synchronization problems consisting in designing a nonlinear feedback unidirectional coupling term for two (possibly chaotic) dynamical systems in order to drive the trajectories of one of them, the slave system, to a reference trajectory or to a prescribed neighborhood of the reference trajectory of the second dynamical system: the master system. If the slave system is chaotic then synchronization can be viewed as the control of chaos; namely the coupling term allows to suppress the chaotic motion by driving the chaotic system to a prescribed reference trajectory. Assuming that the entire vector field representing the velocity of the state can be modified, three different methods to define the nonlinear feedback synchronizing controller are proposed: one for each of the treated problems. These methods are based on results from the small parameter perturbation theory of autonomous systems having a limit cycle, from nonsmooth analysis and from the singular perturbation theory respectively. Simulations to illustrate the effectiveness of the obtained results are also presented.

**Keywords.** Synchronization, nonlinear feedback, chaotic systems.

**AMS (MOS) subject classification:** 34C28, 93C15, 93D09, 93D20

## 1 Introduction

In recent years in the literature on dynamical system analysis a considerable attention has been devoted to the problem of synchronization of coupled nonlinear dynamical systems (see e.g. [1], [3], [11], [19], [22], [27]). One of the most effective methods for solving such problem consists in designing a feedback coupling term which drives the trajectories of one of the two systems (the so-called slave system) to a prescribed reference trajectory of the second one (named master system). Examples of such approach can be found, for instance, in [21] where the coupling term is represented by a linear feedback of the tracking error. In [25] a bidirectional linear coupling term is proposed to synchronize two chaotic systems. An approach to synchronization based on the classical notion of observers, when the state is not fully available, can be found in [8] and [15]. In many cases, when one deals with nonlinear