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ON CHAOS OF THE LOGISTIC MAPS

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Abstract. This paper is concerned with chaos of a family of logistic maps. It is first proved that a regular and nondegenerate snap-back repeller implies chaos in the sense of both Devaney and Li-Yorke for a map in a metric space. Based on this result, it is shown that the logistic system is chaotic in the sense of both Devaney and Li-Yorke, and has uniformly positive Lyapunov exponents in an invariant set for a certain parameter interval with a lower bound less than a specific value, at which the unique 3-periodic orbit appears. In addition, it shows the exact parameter range for the existence of an asymptotically stable 3-periodic point, and consequently the exact parameter range for the biggest periodic window, i.e., 3-periodic window, in the period-doubling bifurcation diagram.

Keywords. Logistic map; Chaos; 3-periodic window; Snap-back repeller; Lyapunov exponent.

1 Introduction

The well-known logistic system is given by

$$x_{n+1} = f(x_n), \quad n \geq 0, \tag{1}$$

where $f(x) := r\,x\,(1-x)$ is the logistic map and $r > 0$ is a parameter.

System (1) has been extensively studied for a very long time. The logistic model, first developed by the sociologist and mathematician, P. F. Verhulst, was to describe a population growth with limited resources from year n to year $n+1$. The first term, $r\,x_n$, represents the reproduction tendency that is proportional to the nth-year population and the second term, $1-x_n$, denotes the need of coexistence and the sharing of the limited resources.

System (1) has been used in many books as a prototype of dynamical systems because it is not only one of the simplest nonlinear systems, but also exhibits amazingly rich dynamical phenomena. The global behavior of the process in dependence on the parameter was first studied in 1976 by R. M. May [15]. Over the last three decades, the logistic map and unimodal maps have attracted a great deal of interest from many mathematicians and

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