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## EXTREMAL SOLUTIONS FOR A BOUNDARY VALUE PROBLEM OF Nth-ORDER IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS IN A BANACH SPACE<sup>†</sup>

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**Abstract.** By establishing a comparison result and using the upper and lower solutions, the authors obtain the existence of maximal and minimal solutions for a boundary value problem of nth-order nonlinear impulsive integro-differential equations in a Banach space. **Keywords.** Impulsive integro-differential equation; Comparison result; Upper and lower solutions; Cone theory and partial ordering.

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## 1 Introduction

The theory of impulsive differential equations has been emerging as an important area of investigation in recent years (see [1]). But the corresponding theory for impulsive integro-differential equations in Banach spaces is yet to be developed. Most of the recent works in this area discussed the first-order and second-order equations (see [2-7]). In two recent papers [8] and [9], we discussed the existence of solutions and multiple solutions of nth-order non-linear impulsive integro-differential equations in Banach spaces by using the fixed point theory and the fixed point index theory, respectively. Now, in this paper, we shall investigate the extremal solutions of such equations by means of completely different method, i.e. by establishing a comparison result and using the upper and lower solutions.

Consider the boundary value problem (BVP) for nth-order nonlinear impulsive integro-differential equation of mixed type in a real Banach space

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