Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **12 (2005)** 527-563
Copyright ©2005 Watam Press

PORTFOLIO SELECTION VIA REPLICATOR DYNAMICS AND PROJECTIONS OF INDEFINITE ESTIMATED COVARIANCES

Immanuel M. Bomze[1]

[1]ISDS, University of Vienna
Brünner Straße 72, A-1210 Wien, Austria

Abstract. The simplest model of portfolio selection results in a so-called standard quadratic optimization problem (StQP). StQPs consist of finding maxima of (possibly indefinite) quadratic forms over the standard simplex. For obtaining (local) solutions to StQPs of considerably high quality, replicator dynamics have become an increasingly popular device. By help of a simulation study, this paper assesses superiority of replicator dynamics-based methods over popular local optimization procedures which are based on line search. New theoretical results on the exponential version of replicator dynamics complement the first part (Sections 2 and 3) of this study. Section 4 deals with practical applications in finance: here, the StQPs have a quadratic form which in theory is negative-semidefinite. Seen more practically, covariance matrices modelling risk are not known in advance and therefore must be estimated from noisy data. Some of the estimators used may be indefinite also. This problem is dealt with by various projection strategies which convert the indefinite estimated covariance matrix into a positive-semidefinite one.

Keywords. Local search; quadratic optimization; Markowitz model.

AMS (MOS) subject classification: primary: 90C20; secondary: 91B28

1 Introduction: Portfolio Selection as a Standard QP

The familiar mean/variance portfolio selection problem (see, e.g. [18, 19]) can be formalized as follows: suppose there are n securities to invest in, at an amount expressed in relative shares $x_i \geq 0$ of an investor's budget, where $i \in \mathcal{V} = \{1, \ldots, n\}$. Thus, the budget constraint reads $\sum_i x_i = \mathbf{e}^\top \mathbf{x} = 1$, and