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PERTURBATION ANALYSIS OF HOMOCLINIC STRUCTURE IN ZAKHAROV EQUATIONS

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Abstract. Using perturbation theory, we get a low dimensional representation for the one-soliton case for Zakharov equations, subject to periodic boundary conditions and a symmetry constraint. This low dimensional representation is a dynamical equation having one degree of freedom. We present a quantitative description of the stable one spatial dimensional homoclinic structure, which is responsible for the onset of stochastic motion in higher dimensions. Some features of Zakharov equations are also discussed based on the low dimensional representation.

Keywords. Zakharov Equations,Homoclinic Structure, Perturbation, Simulation, Nearly Integrable System
AMS (MOS) subject classification: 37C29,37K40,37K55.

1 Introduction

In the past three decades, Hamiltonian systems have played an important role in advances achieved in exploring the nonlinear dynamic systems. To study such basic systems, effective mathematical theories have been developed. Among them the Kolmogorov-Arnold-Moser(KAM) theorem [2, 7] can be rated as milestone work. A better understanding of Hamiltonian systems with finite degrees of freedom has been obtained than for those with infinite degrees of freedom. In the later case, success has been obtained only for integrable models. For these models, the inverse scattering transform is a powerful analytic tool, but it does not provide an intuitive understanding of the underlying behavior, and it does not work for non-integrable systems. People are interested in knowing how the coherent regular motion transits to a random irregular motion. The approximate methodology in conjunction with numerical simulation turns out to be a feasible approach to this end [4].

In this paper, a perturbation method is introduced to study the onset of stochastic motion in Zakharov equations [11]. As a prototype of nearly integrable system, Zakharov system is a basic mathematical model for the complex Langmuir turbulence phenomenon in plasma physics: it describes the interaction of the slowly varying envelope of a high-frequency electric field and slow ion density fluctuations. Numerical investigations have been