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GENERALIZED GRADIENTS OF INTEGRAL FUNCTIONALS

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Abstract. Let (T, Σ, μ) be a σ-finite positive measure space, X and $Y = (Y, \mathcal{P})$ be two locally convex spaces, and $L_M^*(T)$ be an Orlicz space. The generalized gradients of integral functional $F(x) = \int_T f(t,x)d\mu, x \in X$ and $F(x) = \int_T f(t,x(t))d\mu(t), x \in L_M^*(T,Y)$, where $L_M^*(T,Y) = \{x : T \to Y| \text{ for each p} \in \mathcal{P}, \text{p}(x(\cdot)) \in L_M^*(T)\}$, are investigated and the results improve those given in Clarke[2, 3].

Keywords. Integral functional, generalized gradient, locally convex space.

1 Introduction

The theory of subgradients or generalized gradients is recognized for its many applications to optimization and differential equations. R. T. Rockafellar [5] defined subdifferential ∂f for convex functions, showed how to characterize $\partial f(x)$ in terms of directional derivatives $f'(x,y)$, and established the rules about subdifferential calculus of finite sums. This branch of convex analysis was developed further in the 1960's and applied to many kinds of optimization problems [cf, 6, 7 for exposition]. F. H. Clarke [2, 3] has extended the theory to non-convex functions that are merely lower semicontinuous and used it to derive necessary conditions for non-smooth, non-convex problems in optimal control and mathematical programming. Hiriart-Urruty[4] also made an important progress in this direction. For Lipschitz functions Clarke[2, 3] characterized generalized gradient $\partial^0 f(x)$ by means of a generalized directional derivative $f^0(x;v)$ and established the rules for subdifferential calculus of continuous sums - generalized gradient of integral functionals. In this paper, we will establish the generalized gradient for integral functionals in the case of locally convex spaces and Orlicz spaces. This improved the results given in Clarke [2, 3].

2 Generalized gradients of Lipschitz functions in locally convex spaces

Let $X = (X, \tau)$ be a complete locally convex space, generated by a family of seminorms $\mathcal{P}$ on X, and let $f : X \to \mathcal{R}$ be a given function. We assume that