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MULTIPLICITY RESULTS OF THE GELFAND TYPE SINGULAR BOUNDARY VALUE PROBLEMS FOR IMPULSIVE DIFFERENTIAL EQUATIONS

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Abstract. We study the existence of multiple positive solutions of a singular boundary value problem for second order impulsive differential equations with a real parameter. The existence phenomenon of solutions for the problem depends on the parameter and we give a full analysis about multiplicity, existence and nonexistence with respect to the parameter. Furthermore, we apply our main result to study the existence and multiplicity of positive radial solution for impulsive semilinear elliptic problems defined on either an annulus or an exterior domain. Proofs are mainly employed by the upper and lower solutions method and the fixed point index argument.

Keywords. Second order impulsive differential equation, singular boundary value problem, semilinear elliptic problem, positive solution, radial solution, upper solution, lower solution, fixed point index.

AMS (MOS) subject classification: 34A37, 34B15, 35J25

1 Introduction

In this paper, we consider the existence of multiple positive solutions of a boundary value problem for the following second order impulsive differential equation

$$(P_\lambda) \qquad \begin{cases} u''(t) + \lambda q(t) f(u(t)) = 0, & t \in (0,1), \quad t \neq t_1 \\ \Delta u|_{t=t_1} = I(u(t_1)) \\ \Delta u'|_{t=t_1} = -\frac{I(u(t_1))}{1-t_1} \\ u(0) = a, \quad u(1) = b, \end{cases}$$

where $a \geq 0$, $b \geq 0$, λ a positive real parameter, $\Delta u|_{t=t_1} = u(t_1^+) - u(t_1)$ and $\Delta u'|_{t=t_1} = u'(t_1^+) - u'(t_1^-)$. Throughout this paper, assume that $q \in C((0,1),(0,\infty))$, $f \in C([0,\infty),(0,\infty))$ and $I \in C([0,\infty),[0,\infty))$ is nondecreasing. We notice that q may be singular at $t = 0$ and/or 1 and denote $f_0 \triangleq \lim_{u\to 0^+} \frac{f(u)}{u}$, $f_\infty \triangleq \lim_{u\to\infty} \frac{f(u)}{u}$.

When the impulse effects are absent, (P_λ) reduces to the classical Gelfand type problem which has been widely studied by Choi [2], Wong [11], Dalmasso [3], Ha and Lee [7], Lee [9] and Stanzcy [10]. Problems on the second