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CONVEXITY OF THE REACHABLE SET OF NONLINEAR SYSTEMS UNDER L_2 BOUNDED CONTROLS

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Abstract. Recently [7, 8] the new convexity principle has been validated. It states that a nonlinear image of a small ball in a Hilbert space is convex, provided that the map is $C^{1,1}$ and the center of the ball is a regular point of the map. We demonstrate how the result can be exploited to guarantee the convexity of the reachable set for some nonlinear control systems with L_2-bounded control. This provides existence and uniqueness of the solution in some optimal control problems as well as necessary and sufficient optimality conditions and effective iterative methods for finding the solution.
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1 Introduction

Convexity plays a key role in functional analysis, optimization and control theory. For instance, if a mathematical programming problem is convex, then necessary optimality conditions coincide with sufficient ones, duality theorems hold and effective numerical methods can be constructed [12]. However, convex problems are just a small island in the ocean of non convex ones.

In the present paper we describe the technique, which is useful for establishing convexity in nonlinear control problems. It is based on the recent result [7, 8], asserting convexity of a nonlinear image of a small ball in a Hilbert space. This result is addressed in Section 2, while all other Sections deal with its applications to nonlinear control systems. Namely, in Section 3 we prove the convexity of the reachable set for nonlinear systems provided that the linearized system is controllable while the control is bounded and small enough in L_2 norm. For linear systems the explicit description of the reachable set with L_2-bounded control goes back to Kalman [4]. Lee and Markus [6] (see also [5]) proved that controllability of the linearized system implies local controllability of the nonlinear system. However we don't know any result on the convexity of the reachable set for nonlinear case; some related ideas can be found in the earlier paper of the author [9]. In Section 4 we apply the statements of the previous Section to get necessary and sufficient