PERIODIC BOUNDARY VALUE PROBLEMS FOR SECOND ORDER DIFFERENTIAL EQUATIONS WITH DELAY AND MONOTONE ITERATIVE METHODS*

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Abstract. In this paper we show that the method of monotone iterative technique is valid to obtain two monotone sequences which converge uniformly to extremal solutions of the periodic boundary value problems for second order differential equations with delay without assuming monotonicity in the nonlinear part.
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1   Introduction

The method of upper and lower solutions coupled with the monotone iteration has been applied successfully to obtain results of existence and approximation of solutions for periodic boundary value problems for first order and second order ordinary differential equations (see [1] and the references therein).

Some attempts have been made to extend these techniques to study periodic boundary value problems of differential equations with delay. In [2,3,4,5], the periodic boundary value problem (PBVP) of first order

$$y'(t) = f(t, y, y_t), \quad y(0) = y(T)$$

is considered.

But as far as the authors know, the method of upper and lower solutions coupled with the monotone iterative has rarely been seen for PBVPs of second order delay differential equations. In this paper, we apply the method to study of PBVPs in this area.