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A SIZE-STRUCTURED MODEL FOR THE NUTRIENT-DRIVEN SELF-CYCLING FERMENTATION PROCESS

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Abstract. Self-cycling fermentation is a computer-aided process used for culturing microorganisms. Applications include sewage treatment and toxic waste cleanup. We consider a model of self-cycling fermentation with nutrient level as the triggering factor. The model is formulated in terms of impulsive ordinary and partial differential equations and refined to include the size of the microorganisms. A threshold is determined in terms of biologically relevant parameters that show that size specific parameters can affect the outcome. The model predicts that either the system fails and the population of microorganisms essentially washes out or, more favourably, the fermentor cycles indefinitely, with one impulse per period, maintaining a positive, though oscillatory, number of cells. However, in any case, the average length and surface area always equilibrate.

Keywords. Self-cycling fermentation, impulsive differential equations, size-structure, moment of impulse, emptying/refilling fraction, physiological efficiency coefficient.

AMS (MOS) subject classification: 34K45, 34K60, 92D25, 92D40, 62P12

1 Introduction

Nutrient driven self-cycling fermentation can be described as follows. A culture of microorganisms is introduced into a tank in which it is assumed there is a single growth limiting nutrient. The cells process the nutrient in order to grow and reproduce. It is assumed that the tank is well-stirred, so that cells and nutrient are distributed uniformly throughout the tank. A probe inserted in the tank measures the nutrient level and relays the information to a computer. When the nutrient level reaches a predetermined tolerance, the computer initiates an emptying and refilling process. A set fraction of the volume of the tank is removed and replaced by an equal volume of fresh medium. Once the fresh medium has been added to the tank, the cells are allowed to process the nutrient until the tolerance is reached once more. The same fraction of the contents is again removed and then replaced with an equal volume of fresh medium. This process is allowed to continue. The process is considered successful if it reaches the threshold within a reasonable (finite) time, indefinitely.