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AVERAGE GROWTH AND EXTINCTION IN A TWO DIMENSIONAL LOTKA-VOLTERRA SYATEM

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Abstract. A nonautonomous two dimensional Lotka-Volterra system of differential equations is considered. An extension of the *principle of competitive exclusion* is obtained in terms of an average growth rate.
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1 Introduction

Consider the system of differential equations

$$\begin{cases} u_1'(t) &= u_1(t)[a_1(t) - b_{11}(t)u_1(t) - b_{12}(t)u_2(t)] \\ u_2'(t) &= u_2(t)[a_2(t) - b_{21}(t)u_1(t) - b_{22}(t)u_2(t)], \end{cases} \tag{LV}$$

where the coefficients a_k and b_{kj}, $1 \leq k, j \leq 2$, are continuous. For a given function $f(t)$ on $[t_0, \infty)$, we define f_M and f_L as

$$\begin{aligned} f_M &= \sup \{f(t) \,|\, t_0 \leq t < \infty\} \\ f_L &= \inf \{f(t) \,|\, t_0 \leq t < \infty\}. \end{aligned}$$

We assume that for $k = 1, 2$, the inequalities

$$0 < a_{kL} \leq a_{kM} < \infty, \tag{1.1}$$

$$0 < b_{kkL} \leq b_{kkM} < \infty, \tag{1.2}$$

$$0 \leq b_{kjL} \leq b_{kjM} < \infty, j \neq k \tag{1.3}$$

hold on $[t_0, \infty)$. We define $M[a_k]$ and $m[a_k]$, $k = 1, 2$, as

$$\begin{aligned} M[a_k] &= \lim_{r\to\infty} \sup \left\{ \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a_k(s)\, ds, \;\; t_2 - t_1 \geq r \right\} \\ m[a_k] &= \lim_{r\to\infty} \inf \left\{ \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a_k(s)\, ds, \;\; t_2 - t_1 \geq r \right\}. \end{aligned}$$