CONVERGENT NORMAL FORMS, SYMMETRIES, AND APPLICATIONS TO BIFURCATIONS

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Abstract. We study the introduction of Poincaré-Dulac normal forms of dynamical systems (or vector fields), and we discuss the problem of the convergence of the normalizing transformations. We show that the presence of linear or nonlinear Lie point-symmetries can guarantee, under suitable assumptions, the convergence. As an application of these ideas, we consider some bifurcation problems.

1   Introduction.

In this paper we want to discuss some results coming from the introduction of normal forms of dynamical systems, in the sense of Poincaré-Dulac [1, 4, 5, 18] (see also [10], where many other references can be found), and some applications concerning essentially the problem of the convergence of the normalizing transformations.

After a short review (just to fix notations and in view of the applications to be considered) of the fundamental results concerning the normal forms approach, with special emphasis to the case where the dynamical system admits some symmetry property (Sect. 2), we start our presentation by discussing the nontrivial interplay existing between the normal form approach and the presence of symmetries. In Sect. 3 we will show how the presence of some symmetry property may force the convergence of the normalizing transformation: this result is obtained by means of a suitable use of the convergence criteria given by Bruno [4, 5]. In Sect. 4, the technique of normal forms and the same criteria of convergence are applied to bifurcation problems, showing indeed that this approach may be used to deduce the existence of a general class of branching solutions, which includes – as a particular case – not only standard Hopf bifurcation, but also multi-periodic solutions, and bifurcations with degenerate eigenvalues in the presence of symmetries.

2   The normalizing transformations.

In this section we will give, for the reader's convenience and in view of the applications to be considered below, a short survey of some useful results about the normal