

PERIODIC ORBITS OF AN HYBRID SYSTEM IN \mathbb{R}^3

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Abstract. Roughly speaking hybrid systems are mixtures of continuous dynamics and discrete events. Hybrid systems can describe some natural phenomena more accurately, which purely continuous or discrete systems cannot achieve. As in the differential systems also in the hybrid systems the periodic orbits play a main role for understanding their dynamics. In this paper we classify analytically all the periodic orbits of an hybrid system in \mathbb{R}^3 .

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1 Introduction

Piecewise linear differential systems are widely used across multiple disciplines, including mechanical engineering, electrical circuit analysis, and notably in control theory, which provides a wealth of models for such systems, see for instance [1, 2, 3, 4].

A particular category of piecewise differential systems are the hybrid systems. The concept of a hybrid system varies depending on the field in which it is used. Broadly speaking, hybrid systems are dynamical systems characterized by the interplay between continuously evolving variables and variables that change discretely, often at specific points in time. A comprehensive overview, including formal definitions and a variety of applications, is provided in [5].

Hybrid systems offer a precise mathematical structure for representing and examining systems in which continuous and discrete behaviors are tightly linked. Understanding these systems involves expanding traditional dynamical systems theory to accommodate discontinuities, switching between modes, and logic-based control rules. As a result, hybrid systems play a key role in contemporary fields such as engineering, robotics, cyber-physical systems, and are gaining growing attention in applied mathematics research.