Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **33 (2026)** 135-143
 http://www.watam.org

MILD SOLUTION FOR TIME FRACTIONAL DEBYE-HÜCKEL SYSTEM IN BESOV-MORREY SPACES

Hassan Khaider[1,*], Achraf Azanzal[2] and Abderrahmane Raji[1]

[1]Laboratory LMACS, FST of Beni-Mellal, Sultan Moulay slimane University, Morocco
[2]Laboratory of Education, Sciences and Techniques - LEST, Higher School of Education and Training Berrechid (ESEFB), Hassan First University, Avenue de l'Université, B.P :218, 26100, Berrechid, Morocco.

[*] Corresponding Author Email: hassankhaider1998@gmail.com

Abstract. This paper is focused on the study of the existence and uniqueness of the Debye-Hückel system (DHS) with a fractional Caputo derivative. By using the Mittag-Leffler operators $\{\mathcal{Q}_\alpha(-t^\alpha\mathbb{A}) : t \geq 0\}$ and $\{\mathcal{Q}_{\alpha,\alpha}(-t^\alpha\mathbb{A}) : t \geq 0\}$ we will prove the mild solution of DHS if the initial data belong to the Besov-Morrey space $\mathcal{N}^{r-2}_{p,\mu,1} \times \mathcal{N}^{r-2}_{p,\mu,1}$.

Keywords. Debye-Hückel system, Mittag-Leffler operators, derivative of Caputo, Besov-Morrey space.

AMS (MOS) subject classification: 35Q35, 35R11, 33E12.

1 Introduction

The Debye-Hückel theory for electrolytic solutions is generalized to planar interfacial geometries [8], the Debye-Hückel system is given by:

$$\begin{cases} \partial_t d = \Delta d - \nabla \cdot (d\,\nabla p) & \text{in } \mathbb{R}^n \times (0,\infty), \\ \partial_t h = \Delta h + \nabla \cdot (h\,\nabla p) & \text{in } \mathbb{R}^n \times (0,\infty), \\ \Delta p = d - h & \text{in } \mathbb{R}^n \times (0,\infty), \\ d(x,0) = d_0(x), \quad h(x,0) = h_0(x) & \text{in } \mathbb{R}^n, \end{cases} \tag{1}$$

where the functions $h = h(x,t)$ and $d = d(x,t)$ denote the hole in electrolytes and densities of the electron, respectively, $p = p(x,t)$ denotes the electric potential, $d_0(x)$ and $h_0(x)$ are initial datum.

The mathematical study of the Debye-Hückel system in the 1980s initially specialized in initial boundary value problems. Significant progress was made in understanding the behavior of the system, including aspects such as global existence, uniqueness, regularity of classical solutions and asymptotic stability of stationary solutions. The analysis made use of the Green's function, Poincaré's inequality and the standard maximum principle for parabolic-type equations, [5, 11].