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ON SOLUTIONS TO NONLINEAR DISCRETE BOUNDARY VALUE PROBLEMS ON INFINITE INTERVALS

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Abstract. The results in this paper provide criteria for the solvability of weakly nonlinear discrete boundary value problems on infinite intervals. The paper also contains a qualitative analysis describing the dependence of solutions on small parameters.

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1 Introduction

In this paper, we study nonlinear discrete boundary value problems on infinite intervals. We establish criteria for the existence of solutions to a wide range of problems, and we provide a qualitative analysis of the solutions resulting from perturbations in both the dynamics and the boundary operator.

We consider boundary value problems of the form

$$x(k+1) - A(k)x(k) = h(k) + \varepsilon f(x(k)) \tag{1}$$

subject to

$$\sum_{k=0}^{\infty} C(k)x(k) = u + \varepsilon \sum_{k=0}^{\infty} S(k)g(x(k)) \tag{2}$$

where f and g are continuously differentiable maps from $\mathbb{R}^n$ into $\mathbb{R}^n$; $A(k), C(k)$ and $S(k)$ are $n \times n$ matrices, $A(k)$ is invertible for each k and $\sum_{k=0}^{\infty} \|C(k)\| < \infty$ and $\sum_{k=0}^{\infty} \|S(k)\| < \infty$.

The corresponding linear boundary value problem

$$x(k+1) - A(k)x(k) = h(k) \tag{3}$$