Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **31 (2024)** 431-445
 http://www.watam.org

ON A CLASS OF LERAY-LIONS TYPE PROBLEM IN MUSIELAK-ORLICZ-SOBOLEV SPACES

Noureddine Moujane[1*], Mohamed El Ouaarabi[2] and Said Melliani[1]

[1]Laboratory LMACS, Faculty of Science and Technics
Sultan Moulay Slimane University, Beni Mellal, BP 523, 23000, Morocco

[2]Fundamental and Applied Mathematics Laboratory
Faculty of Sciences An Chock, Hassan II University, Casablanca, BP 5366, 20100, Morocco

Abstract. In this paper, we show the existence of weak solutions to a class of LerayLions type problem under Dirichlet boundary conditions in the framework of Musielak-Orlicz-Sobolev spaces. The proof of the main result is constructed by utilizing the topological degree for a class of demicontinuous operators of (S_+)-type.

Keywords. Leray-Lions operator, Nonlinear elliptic equation, Musielak-Orlicz-Sobolev space, topological degree methods.

AMS (MOS) subject classification: 35J60, 47J05, 35D30, 47H11.

1 Introduction

The study of nonlinear differential equations in modular spaces is of great interest in several fields and very active in many mathematical physics models, such as the modeling of continuum mechanics, elastic mechanics [36], non-Newtonian fluids [23], image restoration [5], theory of potential [26] and electrorheological fluids [25]. Different directions of development have been initiated in the context of existence theory and when studying the general form of nonlinear partial differential equations, it is important to find an appropriate functional space corresponding to their solutions. For example, the p-Laplacian equations correspond to the classical Sobolev space setting [29], the $p(z)$-Laplacian equations correspond to the generalized Sobolev space [1, 9, 11, 8, 34, 10], the Leray-Lions equations studied in various space settings [30, 32, 19].

In this paper we consider Musielak-Orlicz-Sobolev spaces, which provide the framework for a variety of different function spaces, including classical (weighted) Lebesgue, anisotropic, Orlicz and the generalized Sobolev spaces. For further information, the reader is referred to the articles [27, 33, 17].