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# APPLICATION OF AVERY-TYPE FIXED POINT THEOREMS FOR NABLA FRACTIONAL BOUNDARY VALUE PROBLEMS

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**Abstract.** In this article, we consider the following two-point discrete fractional boundary value problem with constant coefficient associated with Dirichlet boundary conditions

$$\begin{cases} -\big(\nabla^{\nu}_{\rho(a)}u\big)(t) + \lambda u(t) = f(t, u(t)), & t \in \mathbb{N}^{b}_{a+2}, \\ u(a) = u(b) = 0, \end{cases}$$

where $1 < \nu < 2$, $a, b \in \mathbb{R}$, with $b - a \in \mathbb{N}_3$, $|\lambda| < 1$, $\nabla^{\nu}_{\rho(a)}u$ denotes the $\nu^{\text{th}}$ Riemann–Liouville nabla difference of $u$ based at $\rho(a)$, and $f : \mathbb{N}^{b}_{a+2} \times \mathbb{R} \to \mathbb{R}^{+}$.

We make use of Avery–Peterson and Avery–Henderson fixed point theorems on suitable cones and under appropriate conditions on the non-linear part of the difference equation to establish sufficient requirements for at least two and at least three positive solutions to the considered boundary value problem.

**Keywords.** Nabla fractional difference, boundary value problem, Dirichlet boundary conditions, positive solution, existence, fixed point.

**AMS (MOS) subject classification:** 39A12.

## 1 Introduction

Nabla fractional calculus is a branch of mathematics that deals with arbitrary order differences and sums in the backward sense. The theory of nabla fractional calculus is still in its early stages, with the most important contributions appearing in the last decade. Gray & Zhang [23] and Miller & Ross [21] introduced the concept of nabla fractional difference and sum. Atici & Eloe [7] developed the nabla fractional Riemann–Liouville difference operator, began the study of the nabla fractional initial value problem, and established the exponential law, product rule, and nabla Laplace transform in this line. Several mathematicians [4, 5, 6, 7, 24, 25, 13, 16, 14, 17, 15] have contributed to the theory of discrete fractional calculus and as a result of their works, today it has turned into a fruitful field of research in science and engineering. We refer to a recent monograph by Goodrich & Peterson [21]