Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **30 (2023)** 437-448
 http://www.watam.org

OPTIMAL CONTROL OF AN EVOLUTION PROBLEM WITH TIME AND STATE-DEPENDENT MAXIMAL MONOTONE OPERATORS

Soumia Saïdi

LMPA Laboratory,
Department of Mathematics, Faculty of Exact Sciences and Informatics,
Mohammed Seddik Benyahia University, 18000 Jijel, Algeria
E-mail address: soumiasaidi44@gmail.com

Abstract. The current paper focuses on a new class of optimal control problems for perturbed differential inclusions driven by time and state-dependent maximal monotone operators with single-valued perturbations (with mixed control-state). We establish some existence results for this kind of dynamics and optimal solutions to the corresponding minimizing Bolza-type functional.

Keywords. Differential inclusion, maximal monotone operator, pseudo-distance, optimal control.

AMS (MOS) subject classification: 34A60, 47J35, 34G25, 49J52, 49J53.

1 Introduction and formulation of the problem

The optimal control problem (P) to investigate in the present work consists of minimizing the Bolza-type functional

$$J_0[x,u,a] = \psi(x(T)) + \int_0^T l_0(t,x(t),u(t),a(t),\dot{x}(t),\dot{u}(t),\dot{a}(t))dt, \tag{1.1}$$

on the set of feasible controls $(u(\cdot),a(\cdot))$ and the associated solutions $x(\cdot)$ of

$$\begin{cases} -\dot{x}(t) \in A(t,u(t))x(t) + f(a(t),x(t)) \quad \text{a.e. } t \in [0,T], \\ \;\; x(t) \in \mathrm{D}(A(t,u(t)))\; t \in [0,T], \\ (u(\cdot),a(\cdot)) \in W^{1,2}([0,T],\mathbb{R}^{n+m}), \\ u(0) = u_0,\; x(0) = x_0 \in \mathrm{D}(A(0,u_0)). \end{cases} \tag{1.2}$$

The operator $A(t,y) : \mathrm{D}(A(t,y)) \subset \mathbb{R}^n \to 2^{\mathbb{R}^n}$, for any $(t,y) \in [0,T] \times \mathbb{R}^n$, is maximal monotone, whose domain is denoted $\mathrm{D}(A(t,y))$. The perturbation $f : \mathbb{R}^{m+n} \to \mathbb{R}^n$ is a continuous map. The terminal cost functional $\psi : \mathbb{R}^n \to \overline{\mathbb{R}}$ and the running cost $l_0 : [0,T] \times \mathbb{R}^{4n+2m} \to \overline{\mathbb{R}}$ satisfy appropriate assumptions.

Define the set-valued map $F : [0,T] \times \mathbb{R}^{2n+m} \to 2^{\mathbb{R}^n}$ by

$$F(t,z) = F(t,x,u,a) = A(t,u)x + f(a,x). \tag{1.3}$$

The associated Cauchy problem to the differential inclusion (1.2) can be expressed in term of $z \in \mathbb{R}^{2n+m}$ as

$$-\dot{z}(t) \in F(t,z(t)) \times \mathbb{R}^n \times \mathbb{R}^m = G(t,z(t)) \text{ a.e. } t \in [0,T], \tag{1.4}$$