

ON A SYMMETRIC SYSTEM OF DIFFERENCE EQUATIONS OF HIGHER ORDER

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Abstract. In this study we give solutions for the following difference equation system

$$x_{n+1} = \frac{ax_n y_{n-k}}{y_{n-k+1} - \alpha} + \beta, \quad y_{n+1} = \frac{by_n x_{n-k}}{x_{n-k+1} - \beta} + \alpha, \quad n \in \mathbb{N}_0$$

where the parameters a, b, α, β and initial values $x_{-i}, y_{-i}, i = 0, 1, 2, \dots, k$ are non-zero real numbers. We show the asymptotic behavior of the system of equations.

Keywords. Difference Equations, Long Term, System of Difference Equations, Asymptotic Behavior

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1 Introduction and Preliminaries

Despite the fact studying nonlinear difference equations have quite complexity, it is an area of a great recent interest since they appear in biology, finance, economy, physics and so on [1]-[16]. In [4] Haddad et al. studied the equation system:

$$x_{n+1} = \frac{ax_n y_{n-1}}{y_n - \alpha} + \beta, \quad y_{n+1} = \frac{bx_{n-1} y_n}{x_n - \beta} + \alpha, \quad n \in \mathbb{N}_0 \quad (1.1)$$

where the parameters a, b, α, β and initial values $x_{-i}, y_{-i}, i = 0, 1$, are non-zero real numbers. From (1.1) a few work has been done (see [3], [15]).

In this study we investigate following system of the equation

$$x_{n+1} = \frac{ax_n y_{n-k}}{y_{n-k+1} - \alpha} + \beta, \quad y_{n+1} = \frac{bx_{n-k} y_n}{x_{n-k+1} - \beta} + \alpha, \quad n \in \mathbb{N}_0 \quad (1.2)$$

where the parameters a, b, α, β and initial values $x_{-i}, y_{-i}, i = 0, 1, 2, \dots, k$ are non-zero real numbers.

Let's give following well known lemma which be used to prove our theorems [2].