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EXISTENCE OF MULTIPLE POSITIVE SOLUTIONS OF SEMIPOSITONE SINGULAR BOUNDARY VALUE PROBLEMS ON TIME SCALES

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Abstract. In this paper, we consider the existence of multiple positive solutions for the following singular semipositone Dirichlet boundary value problem on time scales:

$$\begin{cases} -y^{\Delta\Delta}(t) = p(t)f(t,\, y^{\sigma}(t)) + q(t), \quad t \in (\rho(a),\, \sigma(b))_{\mathbb{T}}, \\ y(\rho(a)) = 0, \quad y(\sigma(b)) = 0, \end{cases}$$

where $p : (\rho(a),\, \sigma(b))_{\mathbb{T}} \to [0,\, \infty)$ and $f : [\rho(a),\, \sigma(b)]_{\mathbb{T}} \times (0,\, \infty) \to [0,\, \infty)$ are continuous and $q : (\rho(a),\, \sigma(b))_{\mathbb{T}} \to (-\infty,\, \infty)$ is Lebesgue Δ -integrable. By constructing a special cone and using a fixed point theorem, we establish some sufficient conditions for the existence of multiple positive solutions when either f is sublinear or superlinear. The results in this paper are new and generalize the results existing in the literature [9, 25, 26]. Examples are given at the end of the paper to justify our results.

Keywords. Positive solution, singular boundary value problems, semipositone, Green's function, fixed point theorem, cone.

AMS (MOS) subject classification: 34B15, 34B16, 34B18, 34N05, 39A10, 39A13.

1 Introduction

The study of dynamic equations on time scales goes to the seminal work of Stefan Hilger [16] and has received a lot of attention in recent years. Time scales were created to unify the study of continuous and discrete mathematics and are particularly used in differential and difference equations. Many results concerning differential equations carry over quite easily corresponding results for difference equations, while other results seems to be completely different from their continuous counterparts. The study of dynamic equations on time scales reveals such discrepancies and allows us to avoid proving results twice, once for differential equations and once again for difference equations. The general idea is to prove the results for a dynamic equation where the domain of the unknown function is a time scale $\mathbb{T}$, which is a nonempty closed subset of the real numbers $\mathbb{R}$. In this way the results of this paper not only apply to the set of real numbers or set of integers but also to more general time scales such as $\mathbb{T} = h\mathbb{N}$, $\mathbb{T} = q^{\mathbb{N}_0} = \{t : t = q^k, k \in \mathbb{N}_0\}$ with $q > 1$, $\mathbb{T} = \mathbb{N}_0^2 = \{t^2 : t \in \mathbb{N}_0\}$, $\mathbb{T} = \{\sqrt{n} : n \in \mathbb{N}_0\}$ etc. For basic