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## ROBUST OUTPUT STABILITY PROPERTIES FOR NONLINEAR DELAY SYSTEMS

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**Abstract.** Motivated by the regulator theory and adaptive controls, several notions on output stability in the framework of input-to-state stability (ISS) were introduced for finitedimensional systems. It turned out that these output stability notions are intrinsically different, reflecting different manners of how state variables may affect the transient behavior of output variables. In this work, we consider these output stability properties for delay systems. Our main objective is to illustrate how the various notions are related for delay systems and to provide Razumikhin criteria for the output stability properties. The main results are also critical in developing the converse Lyapunov theorems of the output stability properties for delay systems.

**Keywords.** time-delay, nonlinear system, input-to-output stability, robust stability, Razumikhin criteria.

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## 1 Introduction

Since introduced in [32], the ISS theory has become a natural framework for robust stability analysis for systems affected by disturbances and uncertainties. Motivated by applications such as tracking and regulation, adaptive control, and observer design, a set of notions on robust output stability were introduced in [33] and [34]. These notions become especially relevant in a situation when it is only possible or necessary to investigate stability properties for a partial set of state variables, or more generally, functions of state variables. Such a situation frequently arises from applications of regulation, adaptive control design, etc. Stability properties such as stability respect to sets (e.g., closed orbits) and incremental stability can also be phrased as appropriate output stability properties. Some related properties were treated in the past literature as partial stability (see [40]), or stability "in two measures"