Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **30 (2023)** 303–307
 http://www.watam.org

DYNAMICS OF A CLASS OF 3–DIMENSIONAL LOTKA–VOLTERRA SYSTEMS

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Abstract. We provide the complete dynamics of the Lotka–Volterra differential system

$$\dot{x} = x(ay - cz), \quad \dot{y} = y(bz - ax), \quad \dot{z} = z(cx - by),$$

where a, b, c are positive parameters and x, y, z are in the positive octant of R^3. In particular we show that this system is completely integrable, i.e. it has two independent first integrals. Fixing one of these first integrals we obtain invariant triangles in the positive octant of R^3. The dynamics of the system on each one of these invariant triangles is given by an equilibrium point surrounded by periodic orbits, i.e. by a center. In short all the orbits of these system are either equilibrium points, or periodic orbits.

This nonlinear differential system models, under the conservation of mass, a cycle of irreversible autocatalytic reactions between the different states of three macromolecules and allows to describe stable chemical oscillations.

Keywords. Lotka–Volterra system, invariant, global dynamics, phase portrait.

AMS (MOS) subject classification: 34C05, 34C23, 34C25, 34C29.

1. Introduction and statement of the results

In 1910, in the work [16], of A.J. Lotka appeared for the first time the now called Lotka-Volterra systems, and in 1920, also Lotka considered the system

$$\frac{dx}{dt} = x(\alpha - \beta y), \quad \frac{dy}{dt} = y(-\gamma + \delta x),$$

to model the interaction between an herbivorous animal and a plant (see [17]). Here the number of preys and predators are denoted by x and y, while the interaction between the two species is given by the real parameters α, β, γ and δ, which are positive. Later on in 1926 Volterra [21] developed the model of Lotka for explaining the relationship between fish and predatory fish. In 1936, Kolmogorov [8] extended these systems to degree greater than two.

Lotka–Volterra as well as Kolmogorov systems have been generalized for analyzing the dynamics of the interaction among several species, and to model the dynamics in many distinct areas, see for instance [1, 2, 3, 7, 9, 10, 12, 13, 14, 19, 22] and the references therein).

In biochemistry, the pioneering work of Wyman [23] models the autocatalytic chemical reactions. When the law of mass conservation is considered