

ON SYSTEMS OF DIFFERENCE EQUATIONS: CLOSED-FORM SOLUTIONS AND CONVERGENCE

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Abstract. We examine the explicit expression in closed-form for the two families of the p -dimensional system of nonlinear difference equations and derive the convergence of positive solutions. Our theorems generalize the classical results known for one-dimensional difference equations relate to Stević's work (*Journal of Applied Mathematics and Computing*, <https://doi.org/10.1007/s12190-022-01780-5>).

Keywords. Convergence; Periodic solution; System of rational difference equations; Square root.

AMS (MOS) subject classification: 39A05, 39A10 and 39A20.

1 Introduction

Systems of nonlinear difference equations surely constitutes the most significant concept in understanding the behavior of their differential counterparts ([30]). Among them, rational-type formulations remain to pull the greater attention, this is due to their simplicity of use, flexibility and symmetry (see., [2] – [13], [16] – [24], [27], [29]). On the other hand, by construction, a system of nonlinear difference equations is a natural extension of the difference equation, in particular, Stević [23] gave the solutions to the following rational difference equation

$$x_{n+1}^{(1)} = \frac{x_{n-1}^{(1)}}{1 + x_n^{(1)} x_{n-1}^{(1)}}, n \geq 0,$$

while Elsayed [2] gave the form of the solutions of the following system of rational difference equations

$$x_{n+1}^{(1)} = \frac{x_{n-1}^{(1)}}{\pm 1 + x_n^{(2)} x_{n-1}^{(1)}}, x_{n+1}^{(2)} = \frac{x_{n-1}^{(2)}}{\pm 1 + x_n^{(1)} x_{n-1}^{(2)}}, n \geq 0,$$

and many more examples (see., [1] – [19], [25] – [26], [28] – [29], [31]). Now, due to the wonderful results that Stević [23] obtained through the following