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# DETERMINISTIC AND STOCHASTIC ONLINE SOCIAL NETWORK MODELS WITH VARYING POPULATION SIZE

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**Abstract.** In this paper, ordinary differential equation (ODE) and stochastic differential equation (SDE) models are developed to describe the dynamics of online social network user adoption and abandonment. A series of criteria on the uniqueness and positivity of solutions as well as the stability of the user-free equilibrium for both ODE and SDE models are obtained. Numerical simulations are then carried out to verify the qualitative analysis results. Finally, a case study utilizing the historical Facebook daily active user data is conducted to demonstrate the practicality of the models.

**Keywords.** Online social networks, infectious recovery, stochastic differential equations, Lyapunov functions, Numerical simulation.

**AMS (MOS) subject classification:** 34D20, 34D23, 60H10, 91D30, 92D25.

## 1 Introduction

Nowadays, online social networks (OSNs) have greatly changed the way we live our lives by providing important virtual platforms for socialization, entertainment, and commercial activities [1, 2]. Effective utilization of OSNs requires a deep understanding of various OSN characteristics. The OSN user number is undoubtedly an important metric of the popularity of an OSN product, which directly impacts the success, sustainability, and strategic planning of the OSN. The lack of an accurate prediction of OSN users will lead to misjudgments of the commercial environment and potential risks in business decisions, which are particularly crucial for the success of emerging OSN products in a competitive environment.

Motivated by this demand, statistical and machine learning methods have been used to develop predictive models for the OSN users [3, 4]. However, the lack of interpretability and the indistinguishability of correlation and causation are two main limitations of those black-box models. Data-driven mathematical models based on rigor reasoning will be a promising solution to help us understand the OSN user dynamics and avoid the limitations of