

BEHAVIOR OF TWO-DIMENSIONAL COMPETITIVE SYSTEM OF NONLINEAR RATIONAL RECURSIVE SEQUENCE

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Abstract. We investigated the solutions of the fractional system of difference equations

$$H_{n+1} = \frac{H_{n-5}}{\pm 1 \pm H_{n-5} L_{n-2}}, \quad L_{n+1} = \frac{L_{n-5}}{\pm 1 \pm L_{n-5} H_{n-2}}, \quad (1)$$

with the initials $H_{-5}, H_{-4}, H_{-3}, H_{-2}, H_{-1}, H_0, L_{-5}, L_{-4}, L_{-3}, L_{-2}, L_{-1}$ and L_0 such that $H_{-5} L_{-2} \neq \pm 1, H_{-4} L_{-1} \neq \pm 1, H_{-3} L_0 \neq \pm 1, H_{-2} L_1 \neq \pm 1, H_{-1} L_2 \neq \pm 1, H_0 L_3 \neq \pm 1, L_{-5} H_{-2} \neq \pm 1, L_{-4} H_{-1} \neq \pm 1, L_{-3} H_0 \neq \pm 1, L_{-2} H_1 \neq \pm 1, L_{-1} H_2 \neq \pm 1,$ and $L_0 H_3 \neq \pm 1$. Moreover, we studied local and global stability, periodicity and boundedness of solutions.

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1 Introduction

Difference equations have wide applications in various engineering and science disciplines.

A system of equations

$$\begin{aligned} H_{n+1} &= f(H_n, L_n), \\ L_{n+1} &= g(H_n, L_n), \end{aligned} \quad (2)$$

where $n = 0, 1, \dots, (H_0, L_0) \in R, R \subset \mathbb{R}^2, (f, g) : R \rightarrow R$ f, g are function is *competitive* if $f(x, y)$ is non-decreasing in x and non-increasing in y ; and $g(x, y)$ is non-increasing in x and non-decreasing in y .

Elsayed [4] studied the system

$$H_{n+1} = \frac{H_{n-1}}{\pm 1 \pm L_n H_{n-1}}, \quad L_{n+1} = \frac{L_{n-1}}{\pm 1 + H_n L_{n-1}}.$$