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OSCILLATION OF EVEN-ORDER NONLINEAR DIFFERENCE EQUATIONS WITH AN ADVANCED ARGUMENT

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Abstract. This paper is concerned with the oscillatory behavior of solutions to a class of even-order nonlinear difference equations with an advanced argument. Using Kiguradze's lemma and summing averaging technique, some sufficient conditions are established to ensure that the equation considered is oscillatory. New theorems are presented and examples are provided to illustrate all the theoretical results obtained.

Keywords. Advanced; difference equations; even order; nonlinear; oscillation.

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1 Introduction

This paper deals with the oscillatory behavior of solutions of the even-order nonlinear difference equation with an advanced argument

$$\Delta\left(a\left(t\right)\left(\Delta^{n-1}x\left(t\right)\right)^{\alpha}\right)+q\left(t\right)x^{\gamma}\left(\tau\left(t\right)\right)=0. \tag{1.1}$$

Throughout this paper, we always assume that the following conditions are satisfied:

(i) α and γ are the ratios of odd positive integers, $\alpha \geq 1$;

(ii) $\{q\left(t\right)\}$ is a sequence of positive real numbers;

(iii) $\{a\left(t\right)\}$ is a sequence of positive real numbers with $\Delta a\left(t\right) \geq 0$, and

(iv) $\{\tau\left(t\right)\}$ is a monotone increasing sequence of real numbers with $\tau\left(t\right) \geq t$.

Moreover, it is assumed that

$$A\left(t,t_0\right)=\sum_{s=t_0}^{t-1}a^{\frac{-1}{\alpha}}\left(s\right)\rightarrow\infty \; as \; t\rightarrow\infty. \tag{1.2}$$