

BEHAVIOR OF A DISCRETE ECOLOGICAL MODEL

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Abstract.

We study the persistence, boundedness and unboundedness, existence and uniqueness of positive equilibrium point, local and global asymptotic stability, and rate of convergence of the following system of exponential form difference equations:

$$x_{n+1} = \alpha_1 + \beta_1 y_n + \gamma_1 y_{n-1} e^{-x_n}, \quad y_{n+1} = \alpha_2 + \beta_2 x_n + \gamma_2 x_{n-1} e^{-y_n} \quad n = 0, 1, \dots,$$

where initial values x_{-1}, y_{-1}, x_0, y_0 and parameters $\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2$ are positive real numbers. Finally, some numerical examples are given to verify our theoretical results.

Keywords. difference equations; boundedness; unboundedness; persistence; local and global asymptotic stability; rate of convergence.

AMS (MOS) subject classification: 39A10, 40A05

1 Introduction

Difference equations or systems of difference equations play a vital role in the development of different sciences ranging from life to decision sciences (see [1]-[5], [7]-[21], [23], [24], [28] and references cited therein). One of the most important types of difference equation is the exponential form difference equation. These types have many applications in our life. For instance, El-Metwally et al [6] studied the qualitative behavior of the following population model

$$x_{n+1} = \alpha + \beta x_{n-1} e^{-x_n}$$

Papaschinopoulos et al. [25] investigated the asymptotic behavior of the positive solutions of the following systems of difference equations

$$x_{n+1} = \alpha_1 + \beta_1 y_{n-1} e^{-x_n}, \quad y_{n+1} = \alpha_2 + \beta_2 x_{n-1} e^{-y_n}$$

$$x_{n+1} = \alpha_1 + \beta_1 y_{n-1} e^{-y_n}, \quad y_{n+1} = \alpha_2 + \beta_2 x_{n-1} e^{-x_n}$$

A. Q. Khan et al. [22] investigated the qualitative behavior of the positive solution of following system of difference equation

$$x_{n+1} = \frac{\alpha e^{-y_n} + \beta e^{-y_{n-1}}}{\gamma + \alpha x_n + \beta x_{n-1}}, \quad y_{n+1} = \frac{\alpha_1 e^{-x_n} + \beta_1 e^{-x_{n-1}}}{\gamma_1 + \alpha_1 y_n + \beta_1 y_{n-1}}$$