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ASYMPTOTIC BEHAVIOR FOR A VISCOELASTIC PROBLEM WITH ACOUSTIC BOUNDARY CONDITIONS AND VARIABLE-EXPONENT NONLINEARITIES

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Abstract. The main goal of this work is to investigate the existence and uniqueness of the global solution of a mixed problem associated with a nonlinear viscoelastic Kirchhoff equation with Balakrishnan-Taylor damping and nonlinear boundary interior sources with variable exponents and acoustic boundary conditions in a bounded domain. We establish a general stability result for the equation without setting any restrictive growth assumptions on the damping at the origin and weakening the usual assumptions on the relaxation function.

Keywords. Balakrishnan-Taylor damping, Acoustic boundary conditions, global existence; general decay, relaxation function, viscoelastic equation; Lebesgue and Sobolev spaces with variable exponents

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1 Introduction

In this paper, we study the following viscoelastic problem with Balakrishnan-Taylor damping, nonlinear boundary interior sources and acoustic boundary conditions involving the variable-exponent nonlinearities

$$u_{tt} - M\left(|\nabla u(t)|^2\right)\Delta u + \int_0^t g(t-s)\,\Delta u(s)\,ds = |u|^{p(x)-1}u \text{ in } \Omega \times (0,+\infty), \tag{1}$$

$$u = 0 \text{ on } \Gamma_0 \times (0,+\infty), \tag{2}$$