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SOLVING NONLINEAR RATIONAL GENERALIZED DIFFERENCE EQUATIONS

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Abstract. In this paper, we will introduce a new technique or method to find the solutions for some difference equations. We will use this technique to deduce the solutions for some nonlinear rational generalized difference equations in the form:

$$x_{n+1} = \frac{x_{n-k}}{(x_{n-k})^{q-1} + \alpha}, \tag{1}$$

where the initial values $x_{-k+l} = a_{-k+l}$, $l = 0, 1, 2, ...k.$ are nonzero real numbers and with $(x_{-k+l})^{q-1} \neq -\alpha$ for $l = 0, 1, 2, ...k.$. Moreover, we have studied the stability and periodicity of solutions for the generalized nonlinear rational difference equations in (1) and for some special cases.

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1 Introduction

Difference equations or discrete dynamical systems [20] is the diverse field that affects almost every branch of pure and applied mathematics.Every dynamical system $a_{n+1} = f(a_n)$ determines a difference equation and vice versa. Recently, there has been great interest in studying difference equations. One reason for this is a necessity for some techniques that can be used in investigating equations arising in mathematical models describing real-life situations in population biology, economic, probability theory, genetics, psychology, etc.

The study of the properties of rational difference equations has been an area of interest in recent years, see the book [5] and references therein (see also [1], [3]).

There are many papers in which difference equations have studied.
Karatas et al.[16] gave that the solution of difference equation

$$x_{n+1} = \frac{x_{n-5}}{1 + x_{n-2}x_{n-5}}.$$