Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **30 (2023)** 121-134
 http://www.watam.org

GENERALIZED q-MITTAG-LEFFLER STABILITY FOR q-HILFER FRACTIONAL ORDER DIFFERENTIAL SYSTEM

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Abstract. This work is on the study of the q-Mittag-Leffler stability of q-Hilfer fractional non-autonomous sysytems using the Lyapunov direct method. Certain sufficient conditions including a new q-Hilfer type fractional comparison principle to assure the q-Mittag-Leffler stability and asymptotic stability are established. An example is provided at the end to emphasis the developed theory.

Keywords. q-fractional calculus, q-Mittag-Leffler, Hilfer fractional derivative, Lyapunov functions, Stability analysis.

AMS (MOS) subject classification: 33E12,37C75,37B25

1 Introduction

Emanated over and above 300 years, the study of fractional Calculus in many fields of science and engineering are still at its beginnings. With its deep applications, the fractional calculus has set its foot into research and testing new ideas on real data. Especially in the field of control theory, physics, image processing, biology, etc, fractional calculus plays a vital role see [20] for example. Though there are many fractional derivative operator, many authors focused mainly on the two iconic Riemann-Liouville and Caputo fractional derivative, see for instance [13, 14, 18].

In 2002 Hilfer [12] generalized Riemann-Liouville operator, later called as Hilfer derivative. The Hilfer fractional derivative operator which is a two element, family of operator denoted by ${}_{t_0}D_t^{\mu,\nu}$, where μ is called order and ν is called the type that enables bridging between the Riemann-Liouville and the Caputo derivative see [11]. Furati et al. [9] and Gu and Trujillo [11] proved the existence and uniqueness of an initial value nonlinear fractional differential equation involving Hilfer fractional derivative

$$\begin{cases} D_{0+}^{\mu,\nu} y(x) & = g(x, y), \\ I_{0+}^{(1-\mu)(1-\nu)} y(0) & = y_0, \end{cases}$$