GLOBAL EXISTENCE AND GENERAL DECAY FOR A DELAYED FLEXIBLE STRUCTURE WITH SECOND SOUND SUBJECTED TO NONLINEAR DAMPING

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Abstract. In this paper, we consider a non-uniform delayed flexible structure damped by a non-linear dissipation term, where the heat flux is given by Cattaneo’s law. We prove the wellposed of the system using semi-group theory and general stability using multiplier method under suitable assumptions on the weights of the delay, heating effect and material damping and regardless of growth assumption on the nonlinear damping term $f$ at the origin.

Keywords. Decay, flexible structure, semigroups theory, exponential stability, second sound, distributed delay.

AMS (MOS) subject classification: 35L05; 37C75; 93D05; 93D15.

1 Introduction

In this paper, we aim to study the following inhomogeneous delayed flexible structure system of second sound with nonlinear damping

\[
\begin{align*}
  m(x)u_{tt} - (p(x)u_x + 2\delta(x)u_{xt})_x + \eta\theta_x + f(u_t) + \int_{t_1}^{t_2} u(s)u_t(t - s) \, ds & = 0 \\
  \theta_t + \eta u_{tx} + kq_x & = 0 \\
  \tau q_t + \beta q + k\theta_x & = 0,
\end{align*}
\]

(1)

where $u(x, t)$ is the displacement of a particle at position $x \in (0, L)$ and time $t > 0$, $\eta > 0$ is the coupling constant, that accounts for the heating effect, and $\beta, k > 0$, $\theta$ is the temperature of the body, $q = q(x, t)$ is the heat flux and the parameter $\tau > 0$ is the relaxation time describing the time lag in the response for the temperature. $s > 0$ is a real number represents the time delay. $m(x)$, $\delta(x)$ and $p(x)$ are responsible for the non-uniform structure