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INTERFACE DYNAMICS FOR A BI-PHASIC PROBLEM IN HETEROGENEOUS POROUS MEDIA

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Abstract. In this paper, we present an approximate solution of a system coupling flows with different densities in heterogenous porous media. This system concerns immiscible fluids, i.e. fluids that are separated by an abrupt interface. Here we are interested in investigating phase saturation and quantifying the interface evolution during the fluid motion. The numerical procedure is based on a finite volume scheme of a specific type. The novelty in this work is, first the derivation of a simplified model without any regularisation, and secondly the use of an efficient numerical scheme that does not require any stabilisation technique, likewise finite difference or element methods.

Keywords. Seawater intrusion problem, Heterogeneous Porous Media, Finite Volumes, Numerical Analysis.

MSC2020-Mathematics Subject Classification: 65N08; 65N12, 65N50; 76T06.

1 Introduction

Fluid flows in an heterogeneous porous medium have attracted attention to many researchers, for example in hydrogeology for exploitation of groundwater. In fact in coastal aquifer, the extraction of groundwater causes saltwater intrusion problems, so that, an interface or transition zone occurs between the freshwater and saltwater. Here, we are interested in the case when the two fluids are immiscible, for more details see [5] and [6]; for the case of miscible fluids the reader is referred to [4]. Many authors have used the finite volume methods to solve such problems, for example see [2] and [3].

To review some previous studies on this issue, we can cite:

- Talibi et al [13] showed the exitence and uniqueness of local solutions for a degenerate seawater intrusion problem.
- Rosier et al in [12] proved the global existence of the solution for a degenerate elliptic-parabolic seawater intrusion problem by using a