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EXISTENCE AND UNIQUENESS RESULTS FOR FRACTIONAL VOLTERRA-FREDHOLM INTEGRO DIFFERENTIAL EQUATIONS WITH INTEGRAL BOUNDARY CONDITIONS

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Abstract. The purpose of this paper is to prove some new existence and uniqueness results for the Caputo fractional integro-differential equations satisfying integral boundary conditions using joint the Arzela-Ascoli's theorem, Leray-Schauder nonlinear alternative, upper and lower solutions, Schauder and Banach fixed point theorems. Furthermore, we adopt some new conditions on the nonlinear terms to demonstrate the realized improvements in our results proved herein. At last, an example is given to illustrate the main results.

Keywords. Caputo fractional derivative, Volterra-Fredholm integro-differential equation, Positive solutions, Upper and lower solutions, Fixed point method.

AMS (MOS) subject classification: 26A33, 34A08, 34B15.

1 Introduction

Differential and integro-differential equations are often more realistic to describe natural phenomena and they arise in many areas of science, engineering and applied mathematics (refer [2, 11, 12, 15, 24, 31], and the cited references). Fractional differential equation is a generalization of ordinary differential equations and integration to arbitrary non integer order. Differential equations with fractional order derivative have recently proven to be strong tools in the modeling of many physical phenomena and also in various field of science and engineering. There has been a significant development in ordinary and partial fractional differential equations with fractional order in recent years; see the monographs of [3, 6, 13, 14, 15, 21, 22, 26, 28, 29]. In the fractional calculus the various integral inequalities plays an important role in the study of qualitative and quantitative properties of solution of differential and integral equations.

Now a days, many authors focus on the development of techniques for discussing the solutions of fractional differential and integro-differential equations. For instance, we can remember the following works: