Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **30 (2023)** 55-74

AN ALGORITHM FOR SOLVING HAMMERSTEIN TYPE EQUATIONS IN REFLEXIVE BANACH SPACES

A. R. Tufa[1], H. Zegeye[2] and O. Daman[1]

[1]Department of Mathematics
University of Botswana, Pvt. Bag 00704, Gaborone, Botswana

[2]Department of Mathematics and Statistical Sciences
Botswana International University of Science and Technology (BIUST), Pvt. Bag 16, Palapye, Botswana

Abstract. Let X^* be the dual space of a real Banach space X, $F : X \to X^*$ and $K : X^* \to X$ be Lipschitz monotone maps. An explicit iterative process is introduced for solving the Hammerstein type equation, $u + KFu = 0$, in reflexive real Banach spaces assuming that the solution exists. Then, strong convergence results are established under appropriate conditions. Many of the existing results in the literature are generalized and improved in this paper.

Keywords. Dual space, Hammerstein type equation, monotone map, reflexive Banach space, Strong convergence.

AMS (MOS) subject classification: 47H05, 47H30, 47J05

1 Introduction

Let X^* be the dual space of a real Banach space X and C be a nonempty subset of X. A map $F : C \to X^*$ is called *monotone* if

$$\langle x - y, Fx - Fy\rangle \geq 0, \forall x, y \in C.$$

We say that the map F is α*-inverse strongly monotone* if there exists $\alpha > 0$ such that

$$\langle x - y, Fx - Fy\rangle \geq \alpha||Fx - Fy||^2_{X^*}, \forall x, y \in C.$$

One can easily observe that an α-inverse strongly monotone map is monotone.

Let $F : X \to X^*$ be a monotone map and $G(F) = \{(x, Fx) : x \in X\}$ be the graph of F. Then F is said to be *maximal* if $G(F)$ is not a proper subset of $G(T)$, for any other monotone map $T : X \to X^*$. That means, a monotone map F is maximal if and only if $u = Fx$, whenever $(x, u) \in X \times X^*$ and $\langle x - y, u - v\rangle \geq 0$ for every $(y, v) \in G(F)$.