

## NON-AUTONOMOUSLY PERTURBED AUTONOMOUS SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

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**Abstract.** For an autonomous system, we consider a non-autonomous perturbation, which is the right-hand side of an autonomous system multiplied by a scalar time-dependent function. It is proved that this perturbation of the autonomous system preserves the qualitative properties of the solutions of the autonomous system, such as the presence of periodic solutions and the stability of solutions in the sense of Lyapunov. These results make it possible to know what kind of perturbation will not affect the qualitative behavior of solutions when modeling real-world processes.

**Keywords.** Periodic solution; equilibrium point; uniform asymptotic Lyapunov stability; limit cycle; reflecting function.

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### 1 Introduction

In the qualitative theory of ordinary differential equations, much attention is paid to the question of the existence, number and location of periodic solutions. This question can sometimes be answered using a reflecting function (see [8]).

Consider system

$$\dot{x} = X(t, x), \quad t \in \mathbb{R}, \quad x \in D \subset \mathbb{R}^n, \quad (1)$$

whose solutions are uniquely determined by the initial conditions. Let  $x = \varphi(t; t_0, x_0)$  be a general solution of the Cauchy form of the system (1), then the function  $F(t, x) := \varphi(-t; t, x)$ , defined in some domain containing the hyperplane  $t = 0$ , is called *the reflecting function* of system (1) (see [8, p. 62]).

If a continuously differentiable function  $F(t, x)$  (or its restriction) is defined in a domain of  $\mathbb{R}^{1+n}$ , which contained the hyperplan  $t = 0$ , and the identity  $F(-t, F(t, x)) \equiv F(0, x) \equiv x$  is hold, then  $F(t, x)$  is a reflecting