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A VARIATIONAL APPROACH TO DISCRETE ELLIPTIC PROBLEMS WITH A WEIGHT

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Abstract. We establish sufficient conditions for the existence of at least three distinct nonnegative solutions for a discrete problem with a positive weight function. Technical approach used in this article is based on the critical point theory and variational method. We also provide an example to illustrate the result.

Keywords. Nonlinear algebraic systems; Discrete problems; Variational approach; Critical point theory.

AMS (MOS) subject classification: 35B38; 39A10.

1. Introduction

The modeling of certain nonlinear problems in many areas of science have led to the rapid advance in the theory of difference equations. For this reason, in the last decade, the study of the existence and non-existence of solutions to discrete problem has been eagerly discussed.

In this article, we consider the discrete boundary value problem

$$(1)\qquad \begin{cases} -\Delta_1(q(c-1,d)\Delta_1 u(c-1,d)) - \Delta_2(q(c,d-1)\Delta_2 u(c,d-1)) \\ = \lambda f((c,d),u(c,d)), \qquad\qquad \forall (c,d)\in[1,M]\times[1,N], \\ u(0,d)=u(M+1,d)=0, \quad \forall d\in[1,N], \\ u(c,0)=u(c,N+1)=0, \quad \forall c\in[1,M], \end{cases}$$

where $\lambda > 0$ is a real parameter and $[1,M] := \{1,2,\dots,M\}$, $[1,N] := \{1,2,\dots,N\}$, and $f : [1,M]\times[1,N]\times\mathbb{R}\to\mathbb{R}$ denotes a continuous function and $q : [0,M]\times[0,N]\to(0,+\infty)$ is a function such that

$$(2)\qquad q(0,d)=0, \quad \forall d\in[1,N], \quad \text{and} \quad q(c,0)=0, \quad \forall c\in[1,M].$$

The forward difference operators are defined by

$$\Delta_1 u(c,d) = u(c+1,d)-u(c,d) \text{ and } \Delta_2 u(c,d) = u(c,d+1)-u(c,d).$$