Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **29 (2022)** 305-376

MATHEMATICAL MODELLING AND NONLINEAR CONTROL OF A REAR-WHEEL-DRIVE VEHICLE BY USING THE NEWTON-EULER EQUATIONS - PART 2 [1]

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Abstract. This is Part 2 of a two part series of works dealing with the mathematical modelling and nonlinear control of a rear-wheel-drive vehicle by using the Newton-Euler equations and the d'Alembert principle. The vehicle considered in Part 1 is subject to a set of holonomic and nonholonomic velocity constraints that are not independent. Thus, in Part 2, the Newton-Euler equations for constrained multibody systems are extended for the case where the velocity constraints may not be independent. The results indicate that in the case of independent velocity constraints the Lagrange multipliers are unique while in the case of dependent velocity constraints the Lagrange multipliers are not unique. In addition, two state space forms of the Newton-Euler equations for constrained multibody systems are derived and denoted by the abbreviations NE1 and NE2. State space form NE1 requires the derivation of the kinematic model of the system while state space form NE2 does not require the kinematic model. The vectors of Lagrange multipliers associated with state space forms NE1 and NE2 are denoted by $\boldsymbol{\lambda}_{NE1}$ and $\boldsymbol{\lambda}_{NE2}$, respectively. A method is proposed to practically compute $\boldsymbol{\lambda}_{NE1}$, $\boldsymbol{\lambda}_{NE2}$, by using the Moore-Penrose generalized inverse. The proposed method yields vectors of Lagrange multipliers $\boldsymbol{\lambda}_{NE1}$, $\boldsymbol{\lambda}_{NE2}$, that have minimum Euclidean norm and is applicable to cases of independent and not independent velocity constraints. The obtained expressions for $\boldsymbol{\lambda}_{NE1}$, $\boldsymbol{\lambda}_{NE2}$, do not have the same form and contain the Moore-Penrose generalized inverse of a different matrix term. By employing additional derivations it is shown that $\boldsymbol{\lambda}_{NE1} = \boldsymbol{\lambda}_{NE2}$. Thus, Part 1 applies the results of Part 2 and employs state space form NE1 in order to derive the kinematic and dynamic models of the vehicle by using all the velocity constraints in their original form.

Keywords. Nonlinear control, Newton-Euler equations, d'Alembert principle, Generalized applied forces, Generalized constraint forces, Geometric constraints, Nonholonomic and holonomic velocity constraints, Independent and not independent velocity constraints, Velocity constraints matrix, Kinematic model, Reduced dynamic model, Lagrange multipliers, State space form, Moore-Penrose generalized inverse, Rear-wheel-drive vehicle, Differential gear-box, Front wheel steering mechanism.

AMS (MOS) subject classification: 70E, 70Q05, 93B, 93C.

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[1] Gratefully dedicated to the memory of our unforgettable and beloved mother Angeliki Frangos (*Αγγελική Φράγκου*), 1926-2016. Constantinos Frangos, Evangelia Frangos.