Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **29 (2022)** 321-345
 http://www.watam.org

BLOW UP, LOWER BOUNDS AND EXPONENTIAL GROWTH TO A COUPLED QUASILINEAR WAVE EQUATIONS WITH DEGENERATE DAMPING TERMS

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Abstract. In this paper, we consider a quasilinear coupled wave equations with degenerate damping terms in a bounded domain. We obtain several results concerning blow up, lower bounds for blow up time and exponential growth for the problem.

Keywords. Blow up, Lower bounds, Exponential growth, Degenerate damping.

AMS (MOS) subject classification: 35B44, 35L05.

1 Introduction

In this paper, we study the following initial-boundary value problem:

$$\begin{cases} |u_t|^j u_{tt} - div\left(\rho\left(|\nabla u|^2\right)\nabla u\right) - \Delta u_{tt} + \left(|u|^k + |v|^l\right)|u_t|^{s-1} u_t = f_1, \\ |v_t|^j v_{tt} - div\left(\rho\left(|\nabla v|^2\right)\nabla v\right) - \Delta v_{tt} + \left(|v|^\theta + |u|^\varrho\right)|v_t|^{r-1} v_t = f_2, \\ u(x,t) = v(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), \ u_t(x,0) = u_1(x), & x \in \Omega, \\ v(x,0) = v_0(x), \ v_t(x,0) = v_1(x), & x \in \Omega \end{cases} \tag{1}$$

where Ω is a bounded domain with smooth boundary $\partial\Omega$ in R^n $(n = 1, 2, 3)$; $s, r \geq 1,\ k, l, \theta, \varrho \geq 0$ and $\rho,\ f_1,\ f_2$ are given functions to be specified later.

Such evolution equations with damping appear in many contexts in material science and physics. For instance, in the context of fluid flows, viscosity effects often arise as damping terms in evolution equations. In addition, in classical mechanics, such as the physical problems of vibrating membranes and strings or shells in elastic media, damping terms reflect the internal energy that is dissipated by the motion (see [2]). Here, $u(x,t)$ and $v(x,t)$