

VARIATIONAL APPROACHES TO EQUATIONS OF $P(X)$ -LAPLACE TYPE OPERATORS WITH VARIABLE EXPONENTS AND NONLINEAR BOUNDARY CONDITIONS

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Abstract. In this paper, under appropriate growth conditions on the nonlinear terms and by applying variational methods and the some recent critical points theorems, we establish the existence of multiple solutions for a nonlinear elliptic equations of $p(x)$ -Laplace type operators with variable exponents, describing precisely a range for the parameter in all the results of existence and multiplicity. Some applications and examples illustrate the obtained results.

Keywords. Variable exponent; Nonlinear boundary conditions; $p(x)$ -Laplace type operators.

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1 Introduction

In the present paper, we are concerned with the existence and multiplicity of weak solutions of nonlinear Neumann boundary problems and variable exponents involving $p(x)$ -Laplace type

$$\begin{cases} -\operatorname{div} [a(x, \nabla u)] + |u|^{p(x)-2}u = \lambda f(x, u), & \text{in } \Omega, \\ a(x, \nabla u) \cdot \nu = \lambda g(x, u), & \text{on } \partial\Omega, \end{cases} \quad (1.1)$$

where $\lambda \in [0, \infty)$, $\Omega \subset \mathbb{R}^N (N \geq 2)$ is a bounded domain with smooth boundary $\partial\Omega$ and ν is the outward normal vector on $\partial\Omega$, $f : \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ and $g : \partial\Omega \times \mathbb{R} \rightarrow \mathbb{R}$ are two Carathéodory functions and $p \in C(\bar{\Omega})$ is the variable exponent and is globally log-Holder continuous on Ω (see Definition