

EXISTENCE OF SOLUTIONS OF A NONLINEAR BOUNDARY VALUE PROBLEM ON THE HALF-LINE

Lin Li¹ and Saeid Shokooh²

¹School of Mathematics and Statistics
Chongqing Technology and Business University, Chongqing 400067, P.R. China
lilin420@gmail.com

²Department of Mathematics, Faculty of Basic Sciences
Gonbad Kavous University, Gonbad Kavous, Iran
shokooh@gonbad.ac.ir (corresponding author)

Abstract. In this article, we establish the existence of at least one non-trivial classical solution for a nonlinear boundary value problem on the half-line. We use the variational methods for smooth functionals defined on reflexive Banach spaces.

Keywords. Classical solution; nonlinear problems; critical point theory; variational methods.

AMS (MOS) subject classification: 34B15, 34B40, 35J75, 58E05

1 Introduction

Boundary value problems on the half-line occur naturally in the study of radially symmetric solutions of nonlinear elliptic equations [11] and various physical phenomena [14], such as unsteady flow of gas through a semi-infinite porous media, the theory of drain flows, plasma physics and in determining the electrical potential in an isolated neutral atom. For some general and recent works on the theory of boundary value problems of differential equations on the half-line, we refer the reader to [4, 10, 15, 16, 21]. Gomes and Sanchez [10], by using a variational method and critical point theory, studied the existence of solutions for the following equations:

$$\begin{cases} -u''(t) + p(t)u(t) = q(t)h(u(t)) & \text{a.e. } t \in [0, \infty) \\ u(0) = 0 \\ \lim_{x \rightarrow \infty} u(x) = 0. \end{cases}$$

By establishing special Banach spaces, the authors proved the existence of solution for this problems. However, the general theory on the semi-infinite interval is not well developed and most of the known results require rather