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# GENERAL AND OPTIMAL DECAY RESULTS OF A SINGULAR VISCOELASTIC WAVE EQUATION WITH AN INTEGRAL CONDITION

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**Abstract.** In this paper, we consider a viscoelastic wave equation with a Bessel operator and a weighted integral condition and establish a very general decay result, using a non traditional multiplier method and taking advantage of some properties of the convex functions. This result improves many other results in the literature.

**Keywords.** Integral condition; viscoelastic equation; general decay; Bessel operator; relaxation functions

**AMS (MOS) subject classification:** 35L05, 35B35, 35B40, 35L81

## 1 Introduction

In this work, we consider the following viscoelastic problem

$$\begin{cases} u_{tt}(x,t) - \frac{1}{x}(xu_x(x,t))_x + \int_0^t g(t-s)\frac{1}{x}(xu_x(x,s))_x ds = 0, & x \in (0,l),\ t > 0 \\ u_x(l,t) = 0, \qquad \int_0^l xu(x,t)dx = 0, & t \geq 0 \\ u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), & x \in (0,l), \end{cases} \tag{1}$$

where $l < +\infty$, $g$ is a positive nonincreasing function satisfying some conditions to be specified later and $u_0, u_1$ are given data. This problem models the motion of a two-dimensional viscoelastic body on a disc centered at the origin in the radial solution case. So, if we consider the following equation

$$u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau = 0$$

in the polar coordinates, the Laplacian is given by

$$\Delta u = \frac{1}{r}(ru_r)_r + \frac{u_{\theta\theta}}{r^2}.$$

When we look for radial solutions (in this case $u_0$ and $u_1$ must be radial), we obtain (1) with one Dirichlet or Neumann boundary conditions. These