

## THRESHOLD ANALYSIS FOR ONE-AND-TWO DIMENSIONAL DISCRETE SCHRÖDINGER OPERATOR WITH NON-LOCAL POTENTIAL

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**Abstract.** We consider the discrete Schrödinger operator  $H_{\lambda\mu}$  on the lattice  $\mathbb{Z}^d$ ,  $d = 1, 2$ , with a non-local potential constructed via the composition of Dirac delta function and the shift operator. The existence of lower eigenvalue behaviors on the parameters  $(\lambda, \mu) \in \mathbb{R}^2$  of the operator is explicitly derived.

We investigate the threshold resonance and embedded eigenvalue problem on the manifold in  $\mathbb{R}^2$ , being a downward parabola, on which the lowest eigenvalue of the operator gets absorbed into the essential spectrum.

We show that if the lowest eigenvalue is absorbed into the essential spectrum, it can turn to a threshold resonance ( resp. super-threshold resonance) at the left  $\lambda$ -intercept of the parabola, while to the regular at the other points of the parabola when  $d = 2$  (resp  $d = 1$ ).

**Keywords.** Discrete Schrödinger operators, threshold resonance, super-threshold resonance, eigenvalues, lattice

**AMS (MOS) subject classification:** Primary: 81Q10, Secondary: 39A12, 47A10, 47N50.

## 1 Introduction

The spectrum of the discrete Schrödinger operators has attracted considerable attention for both combinatorial Laplacians and quantum graphs; for some recent summaries see [5, 8, 3, 7, 4, 17, 11] and the references therein. Particularly, eigenvalue behavior of Schrödinger operators on the lattice are discussed in e.g. [1, 6, 2, 10, 13, 14] and are briefly discussed in [9, 12, 10, 16] when potentials are Dirac delta function.

In the present paper, we consider the one and two dimensional discrete Schrödinger operator  $H_{\lambda\mu}$  with non-local potential constructed by the composite of the Dirac delta function and shift operator, depending on two parameters  $\lambda, \mu \in \mathbb{R}$  and the element  $x_0 \in \mathbb{Z}^d$ . Our main goal is to investigate the lower eigenvalues, threshold resonances and embedded eigenvalues of this