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THE POTENTIAL FUNCTION PROBLEM FOR THE KLEIN-GORDON EQUATION

A Mathebula[1] and Sameerah Jamal[2]

[1]School of Mathematics,
University of the Witwatersrand, Johannesburg, South Africa.

[2]School of Mathematics,
University of the Witwatersrand, Johannesburg, South Africa.

Abstract. Several spaces which are decomposable are investigated in relation to the collineations and Lie symmetries of the Klein-Gordon equations. Nine classes of solutions are presented, including their conformal vectors. Some of the spaces represent perfect fluids or vacuum spaces. For each vector field, explicit expressions are obtained to define the potential function embedded into the Klein-Gordon equation.

Keywords. Klein-Gordon equation; conformal vectors; potential functions; reducible spaces; vector fields.

AMS (MOS) subject classification:37L20; 35K57; 70G65; 58J72.

1 Introduction

The solution of differential equations is vastly simplified by the existence of point symmetries [13, 2, 6, 7, 8, 3, 18]. One particular set of equations that has showcased this idea is that of the field equations in General Relativity. Knowledge of symmetries in this area is useful in classifying spacetimes by the structure of the Lie algebra spanned by these symmetries. The study of isometries and homotheties in particular is of considerable interest. In a related study, the geometric nature of the symmetries of Klein-Gordon equations have been recently explored [9, 10]. Klein-Gordon equations feature in numerous physical applications, including solid state physics, quantum field theory and nonlinear optics.

The main purpose of this paper is to investigate the conformal symmetries in relation to a Klein-Gordon equation in a special, but important class of spacetimes, namely conformally decomposable 2+2 spaces. Decomposable or reducible spacetimes are characterized by the existence of certain covariantly constant tensor fields or, if its holonomy group is non-degenerately reducible [4]. A 2+2 conformally decomposable space admits a rank-2 symmetric, covariantly constant tensor field and its manifold is the product of two two-dimensional manifolds [12]. More concisely, a spacetime (M, g) is said to be