

ON A THREE DIMENSIONAL HIGHER ORDER SYSTEM OF DIFFERENCE EQUATIONS

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Abstract. In this work, we derive the solutions form of the following three-dimensional system of nonlinear difference equations of higher-order

$$x_{n+1} = \frac{x_{n-k+1}^p y_n}{\alpha y_{n-k}^p + \beta y_n}, \quad y_{n+1} = \frac{y_{n-k+1}^p z_n}{a z_{n-k}^p + b z_n}, \quad z_{n+1} = \frac{z_{n-k+1}^p x_n}{A x_{n-k}^p + B x_n}, \quad n \in \mathbb{N}_0, \quad p, k \in \mathbb{N}.$$

where the parameters $\alpha, \beta, a, b, A, B$ and the initial values $x_{-i}, y_{-i}, z_{-i}, i \in \{0, 1, \dots, k\}$ are non-zero real numbers. The behavior of the solutions of our system with $p = 1$ is provided in details.

Keywords. Systems of difference equations; solution form; boundedness; limiting properties; periodicity..

AMS (MOS) subject classification: 39A10, 39A23, 40A05.

1 Introduction

Difference equations or their systems play an important role in applications and are widely used in modeling discrete real phenomena. Discretizations procedures, like for example Euler Method, Runge-Kutta..., for continuous models expressed using derivatives leads to difference equations or systems of difference equations. This fact explain the interest accorded to this hot topics, in fact in the last two decades, many papers on these equations and their systems have been published (see eg. [1] - [22]).

One of the basic nonlinear difference equation is

$$x_{n+1} = \frac{x_{n-1} x_n}{x_n + x_{n-2}}, \quad n \in \mathbb{N}_0. \quad (1.1)$$