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EXISTENCE OF POSITIVE SOLUTIONS OF A HAMMERSTEIN INTEGRAL EQUATION USING THE LAYERED COMPRESSION-EXPANSION FIXED POINT THEOREM

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Abstract. In this paper, we show the existence of a positive solution of a Hammerstein integral equation under certain conditions on the kernel. We apply the recent Layered Compression-Expansion Fixed Point Theorem. Finally, we provide corollaries to help in application of the main results and present an example.

Keywords. Hammerstein equation, layered compression-expansion, fixed point, positive solution, symmetric solution

AMS (MOS) subject classification: Primary 45G10, Secondary 34B15, 34B27, 47H10

1 Introduction

Let $T_1, T_2 \in \mathbb{R}$ with $T_1 < T_2$. Consider the Hammerstein integral equation

$$x(t) = \int_{T_1}^{T_2} G(t,\tau) f(x(\tau))\, d\tau, \quad t \in [T_1, T_2], \tag{1.1}$$

where $f \in C([0,\infty),[0,\infty))$ and $f = f_\uparrow + f_\downarrow$ is the sum of monotonic increasing and decreasing functions, respectively.

Under certain conditions, we show the existence of positive solutions to (1.1). Our approach is to use the Layered Expansion-Compression Fixed Point Theorem [3] with an appropriately defined cone. A couple of additional assumptions yield the existence of positive symmetric solutions to (1.1). Here, we impose symmetry on the problem and investigate the half interval. Once we obtain the existence of a positive solution on the half interval, we extend to the full interval using symmetry.