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UPPER AND LOWER SOLUTIONS METHOD FOR POSITIVE SOLUTIONS OF ψ-CAPUTO FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we consider a new class of boundary value problem for a nonlinear generalized fractional differential equation of the form

$$\begin{cases} {}^{C}D_{0+}^{\alpha,\psi}u(t) + f(t,u(t)) = 0, \quad t \in [0,1], \\ u'(0) = 0, \quad u(0) + \rho u(1) = \int_0^1 g(s,u(s))ds. \end{cases}$$

where $1 < \alpha < 2$ is a real number, $0 < \rho < 1$, ${}^{C}D_{0+}^{\alpha,\psi}$ is the generalized Caputo fractional derivative, $f, g : [0,1] \times \mathbb{R}^+ \to \mathbb{R}^+$ and $\psi : [0,1] \to \mathbb{R}^+$ are appropriate function. Using the properties of the Green function, we find the corresponding Green's function and prove their positivity under appropriate assumptions. Moreover, the existence and uniqueness of positive solutions to the proposed problem are investigated through the fixed point theory in a cone and constructing the upper and lower solutions of control function.

Keywords: fractional differential equations; Green function; existence; upper and lower solutions; fixed point theorem.

1 Introduction

The fractional calculus (FC) is approximately 300 years old which is a generalization of classical calculus as it deals with the non-integer order. One can discover that there are many definitions of fractional derivatives that have been investigated in the literature. e.g., we refer here to the most well-known types such as Reimann-Liouville, Caputo, Hilfer, Hadamard, and Katugampola derivative, and many others. The best way to deal with an assortment of