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ABSOLUTELY CONTINUOUS INVARIANT MEASURES FOR POSITION DEPENDENT RANDOM MAPS OF PIECEWISE REAL-ANALYTIC MAPS ON THE PLANE

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Let $\{\tau_1, \tau_2, \cdots, \tau_K\}$ be a collection of piecewise real analytic maps on a real analytic partition $\mathcal{P}$ of a bounded region $S \subset \mathbb{R}^2$. Let $\{p_1, p_2, \cdots, p_K\}$ be a collection of position dependent probabilities which are piecewise C^1 on $\mathcal{P}$. We consider position dependent random maps $T = \{\tau_1, \tau_2, \cdots, \tau_K; p_1, p_2, \cdots, p_K\}$. In this paper, we prove the existence of absolutely continuous invariant measures of T.

Keywords. Position dependent random maps on the plane; Absolutely continuous invariant measures; Piecewise real-analytic maps on the plane; The Frobenius-Perron operator;

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1 Introduction

A random map T on a set X is a dynamical system which consists a number of transformations from X into itself, where the process switches from one map to another according to fixed probabilities [9] or, more generally, position dependent probabilities [6] and applied in each iteration of the process. The existence and properties of invariant measures of dynamical systems reflect their long time behavior and play an important role in understanding their statistical properties and chaotic nature [4]. Invariant measures which are absolutely continuous with respect to Lebesgue measure are physical measures, because computer simulations of orbits of the system reveal only invariant measures which are absolutely continuous with respect to Lebesgue measure [4]. Random maps have applications in many areas of sciences and engineering such as in the study of fractals [3], in modeling interference effects in quantum mechanics [5], in computing metric entropy [11], and in forecasting the financial markets [1].

In 1984, Pelikan [9] proved the existence of absolutely continuous invariant measures for random maps on the unit interval with i.i.d. probabilities.