

THE SOLUTION AND DYNAMIC BEHAVIOR OF SOME DIFFERENCE EQUATIONS OF FOURTH ORDER

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Abstract. Difference equations have been recently utilized to model some natural phenomena appearing in various nonlinear sciences. Thus, constructing the exact solutions of such equations is a significantly important area for scientists. The essential purpose of this article is to introduce the solution and the global attractivity of the following difference equations:

$$x_{n+1} = \frac{\alpha x_n x_{n-3}}{\beta x_{n-3} + \gamma x_{n-2}}, \quad x_{n+1} = \frac{\alpha x_n x_{n-3}}{-\beta x_{n-3} - \gamma x_{n-2}}, \quad n = 0, 1, \dots,$$

where the constants α , β and γ are positive real numbers and the initial conditions x_{-3} , x_{-2} , x_{-1} and x_0 are arbitrary non-zero real numbers. We also depict some 2D figures under certain conditions for the obtained results.

Keywords. stability, periodicity, boundedness, global attractor, difference equations.

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1 Introduction

In this work we will present a significant study about the solution behaviour, boundedness and solving some specific forms of the following difference equations:

$$x_{n+1} = \frac{\alpha x_n x_{n-3}}{\beta x_{n-3} + \gamma x_{n-2}}, \quad x_{n+1} = \frac{\alpha x_n x_{n-3}}{-\beta x_{n-3} - \gamma x_{n-2}}, \quad n = 0, 1, \dots, \quad (1)$$

where the initial conditions x_{-3} , x_{-2} , x_{-1} and x_0 are arbitrary non zero real numbers and α , β , γ are positive arbitrary constants.

In recent years, enormous number of researchers and scholars has put a lot of effort into investigating nonlinear difference equations. Specifically, they